Squaring the circle

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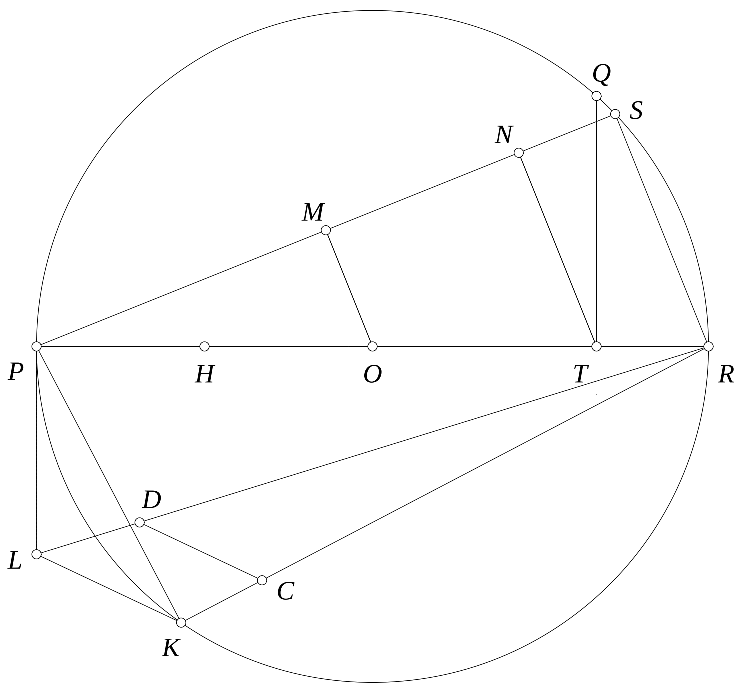
Let PQR be a circle with center O, of which a diameter is PR. Bisect PO at H and let T be the point of trisection of OR nearer R. Draw TQ perpendicular to PR and place the chord $RS = TQ$.
Join PS, and draw OM and TN parallel to RS. Place a chord $PK = PM$, and draw the tangent $PL = MN$. Join RL, RK and KL. Cut off $RC = RH$. Draw CD parallel to KL, meeting RL at D.



Then the square on RD will be equal to the circle PQR approximately. For

$$RS^2 = \frac{5}{36}d^2,$$

where d is the diameter of the circle. Therefore

$$PS^2 = \frac{31}{36}d^2.$$

But PL and PK are equal to MN and PM respectively. Therefore

$$PK^2 = \frac{31}{144}d^2, \quad \text{and} \quad PL^2 = \frac{31}{324}d^2.$$

Hence

$$RK^2 = PR^2 - PK^2 = \frac{113}{144}d^2,$$

and

$$RL^2 = PR^2 + PL^2 = \frac{355}{324}d^2.$$

But

$$\frac{RK}{RL} = \frac{RC}{RD} = \frac{3}{2}\sqrt{\frac{113}{355}},$$

and

$$RC = \frac{3}{4}d.$$

Therefore

$$RD = \frac{d}{2}\sqrt{\frac{355}{113}} = r\sqrt{\pi}, \quad \text{very} \quad \text{nearly}$$

Note: If the area of the circle be 140,000 square miles, then RD is greater than the true length by about an inch.